St Anne's Division Calculation Policy

Date approved:	19th February, 2024
Date of review:	Summer Term 2025

Rationale:

This policy lays out the expectations for written calculations using division and has been created to support the teaching of a mastery approach to mathematics. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across EYFS, KS1 and KS2.

A Mastery Approach:

A mastery approach to learning involves the following five "big ideas" of effective maths teaching:

Coherence -	a coherent learning progression offering deep and connected understanding
Representation and Structure	concrete, pictorial and abstract representations are carefully structured to help pupils "see the maths"
Mathematical Thinking -	looking for patterns and relationships, making connections, conjecturing, reasoning and generalising, communicating ideas using precise vocabulary
Fluency - - -	efficient, accurate recall of key number facts and procedures, allowing pupils to move between different contexts and representations, choosing strategies
Variation - - -	conceptual variation presents different representations of key features, while procedural variation presents different ways of proceeding through the learning journey (via scaffolding and support, etc)

Concrete – Pictorial – Abstract:

Mathematical understanding is developed through use of representations that are initially concrete (e.g. counters, multilink cubes, dienes, etc), and then pictorial (e.g. part-whole models, place value columns with images of counters in them, etc) to then facilitate abstract working (e.g. formal written methods).

This policy is a guide through an appropriate progression of representations. If at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate. As children move through the different stages, representations should be modelled alongside each other to ensure a secure understanding is maintained. Children should only move onto the abstract method when they have a secure understanding of the concept through an appropriate concrete and pictorial representation. This policy should be used in conjunction with the St Anne's Mathematics policy and St Anne's Mental Calculation policy. Teachers are also encouraged to refer to the NCETM Ready-To-Progress Criteria resources in ascertaining when children are ready to move on to new learning.

Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice guidance aimed at informing and enhancing teaching across all year groups. Many of these ideas come from the NCETM's Calculation Guidance document (published October 2015) and the White Rose Calculation Guidance (published in the academic year 2020-2021), which is intended to sit alongside a school's calculation policy.

Please note that while this list of representations is illustrative of the representations children will be exposed to and work with, it is not intended to be exhaustive and children will also encounter other representation methodologies that are not listed here.

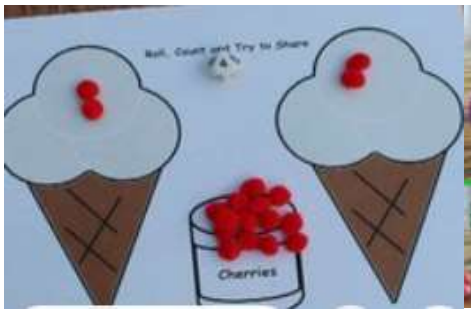
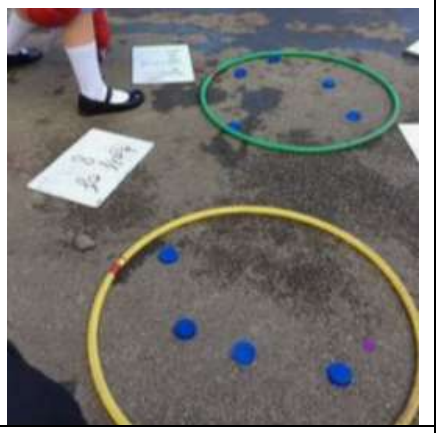
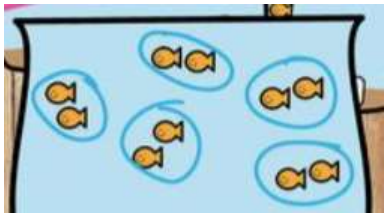
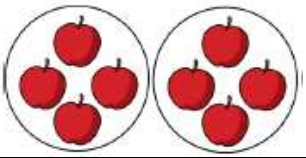
Vocabulary:

Children will continually recap vocabulary learned in previous years to ensure that their understanding and usage of the terminology is fully developed, broad and specific in application. Vocabulary from previous years is included in each year group's columns in black, while new vocabulary that may not have been previously encountered is in green. Teachers are encouraged to check this list of vocabulary at the beginning and end of a relevant unit to ensure that they are modelling the full breadth and depth of vocabulary to the children, and that the children are using it in their verbal and written responses accurately and confidently.

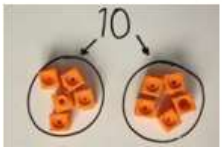
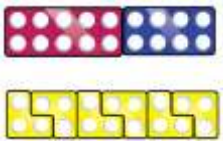
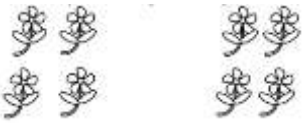
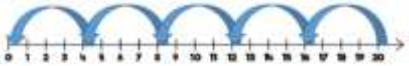
Please see appendix 4 for notes on precise vocabulary, and for a comprehensive glossary, please see the separate document "NCETM Maths Glossary KS1-KS3" which is saved in PDF format with our calculation policies in the shared area.

Contents:

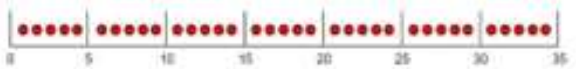
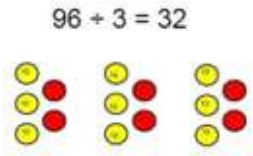
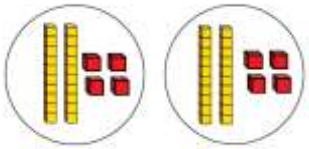
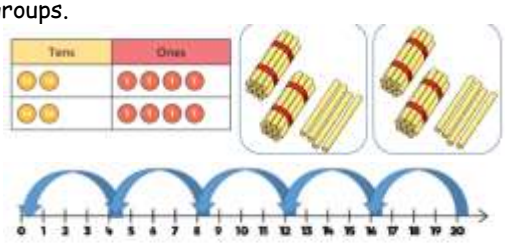
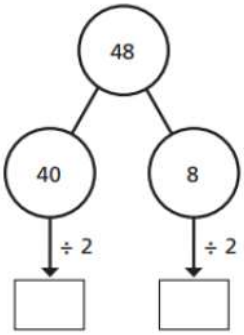
Division in Reception

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Share objects into equal groups. Recognise patterns.	groups of, grouping, sharing, share, shared, ___ each, equal, equally, same size, same amount	**Concrete:** Children will use various objects and counters to represent dividends, which they can then share out into equal groups one at a time to find quotients:   Children will also use hoops to share objects into equal groups (right).  **Pictorial:** Children will draw circles around pictures to share them into groups of set amounts in order to find how many groups of that size they can make (right).   Children will also have set groups to share objects into, as above in the concrete methods (left). **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children may begin to use some written method of recording, such as sentences like: *In each group I have 3 apples.* *Each bear has 5 biscuits.* *I have 4 equal groups.* They may also begin to jot down numbers by their groups of pictures, etc.

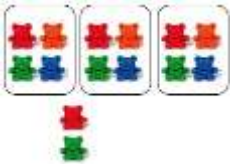
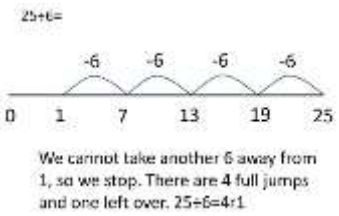
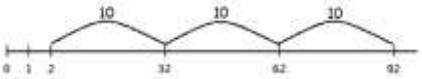
Division in Year 1

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	groups of, grouping, sharing, share, shared, ___ each, **in pairs,** **in ___s,** equal, equally, same size, same amount, **division, divide, divided by, dividing, divided into**	**Concrete:** Children will use various objects and counters, including place value cubes, interlocking cubes and Numicon, to represent dividends and then share these amounts and numbers into equal groups (right).    I have 10 cubes; can you share them equally in 2 groups? Children may also use Numicon to support their understanding of grouping.  They make the dividend, then place the Numicon piece that represents the divisor on top, tessellating it until they make the same amount, then count how many there are altogether to find the quotient (left). **Pictorial:** Children will use pictures or shapes, as well as number lines, to share quantities into groups.   5 × 3 = 15 3 × 5 = 15 15 ÷ 3 = 5  20 ÷ 4 = 5 Note: Counting back on number lines reinforces the concept that the quotient will be smaller than the dividend, making the process of division distinct from the process of multiplication, however children can also count up on number lines rather than back, as long as their understanding of the process is fully secure before doing so. **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children will record divisions as number sentences: *Share 9 buns between 3 people.* 9 ÷ 3 = 3

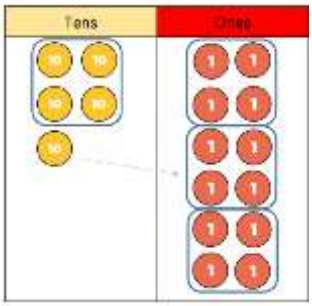
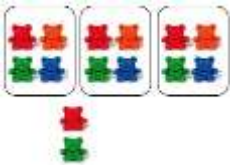
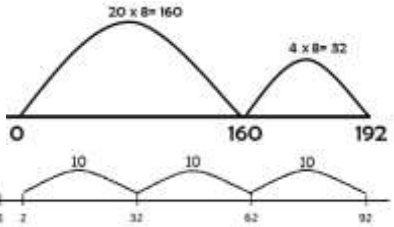
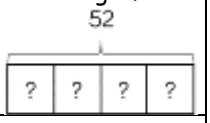
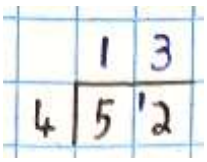
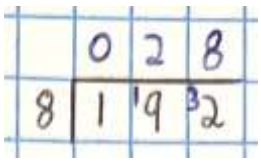
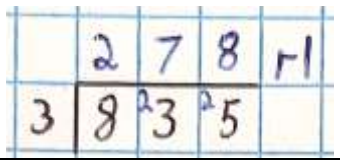
Division in Year 2

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs. Show that division of one number by another cannot be done in any order (not commutative). Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	groups of, grouping, sharing, share, shared, ___ each, in pairs, in ___s, equal, equally, same size, same amount, **division, divide, divided by, dividing, divided into, left (over), array**	**Concrete:** Children will use various objects and counters, including place value cubes, interlocking cubes and place value counters, to represent dividends and then share these amounts and numbers into equal groups.     **Pictorial:** Children will use pictures and base-10 representations, as well as number lines, to share quantities into groups.   20 ÷ 4 = 5 Note: Counting back on number lines reinforces the concept that the quotient will be smaller than the dividend, making the process of division distinct from the process of multiplication, however children can also count up on number lines rather than back, as long as their understanding of the process is fully secure before doing so. **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children will use part-whole models to support them in partitioning dividends into tens and ones (right).  They will also record divisions as number sentences, such as: *Divide 28 into 7 groups.* *How many are in each group?* 28 ÷ 7 = 4

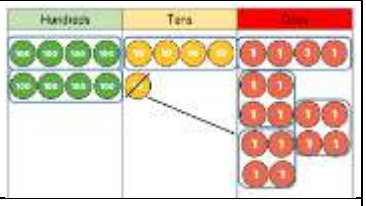
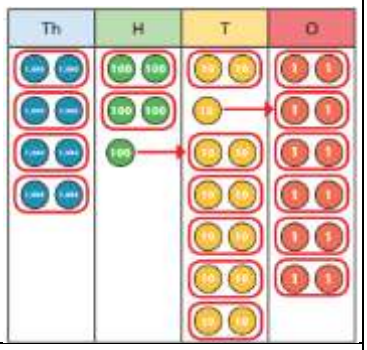
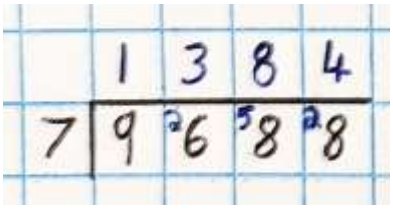
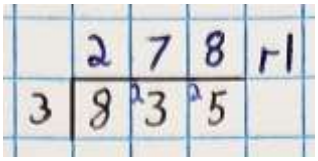
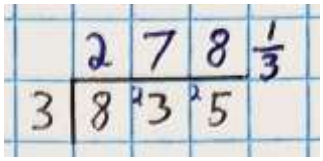
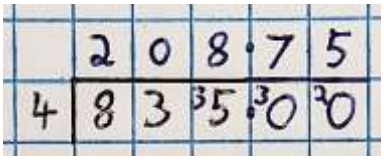
Division in Year 3

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Recall and use division facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for division using the multiplication tables that they know, using mental and progressing to formal written methods. Solve problems, including missing number problems, involving division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	groups of, grouping, sharing, share, shared, ___ each, in pairs, in ___s, equal, equally, same size, same amount, division, divide, divided by, dividing, divided into, left (over), array, **guess, estimate, remainder, approximate, approximately**	**Concrete:**  Children will link division to multiplication by creating arrays and finding the number sentences that can be generated as a result, for example (right): 15 ÷ 3 = 5 15 ÷ 5 = 3 5 x 3 = 15 3 x 5 = 15 5 = 15 ÷ 3 3 = 15 ÷ 5 15 = 5 x 3 15 = 3 x 5 Children will also use base 10 representations to divide numbers equally between groups (right).   When finding remainders, children will initially use objects to see how many are left over, for example when dividing 14 by 3 to find the quotient of 4 remainder 2 (left). **Pictorial:**  Children will draw circles to represent the divisor (the amount of groups), then share out the dividend equally between those groups by drawing dots one at a time. (This is often called "Cookie Method", above.) Children will also use number lines when numbers are too large to use the above method efficiently. Note: Counting back on number lines reinforces the concept that the quotient will be smaller than the dividend, making the process of division distinct from the process of multiplication, however children can also count up on number lines rather than back, as long as their understanding of the process is fully secure before doing so.   As understanding of place value improves, children can begin to jump in multiples of 10 to speed up the process, as in 92 ÷ 3 = 30 r2 for example (left).  Children will also use bar models to represent division (right). **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children in Year 3 use number lines bridging the gap between pictorial and abstract, and written division number sentences as their abstract representation. ***Note: Positive integer scaling problems*** *are where two things are in proportion, so affecting one will affect the other in the same way. For example:* *If 6 apples cost £1, then 3 apples cost £1 (we scaled down by 2).* ***Correspondence problems*** *are where two sets of objects or values are related, and all possible combinations must be found. For example:* *There are 12 possible combinations of hats and coats. If I have 3 hats, how many coats do I have? or* *The total number of legs in the room is 600. How many children could there be? How many rats could there be instead? or* *On the planet Vuv there are Zios (which have 3 legs) and Zepts (which have 7 legs). Nico the explorer counted 52 legs. How many Zios and Zepts could there be?*

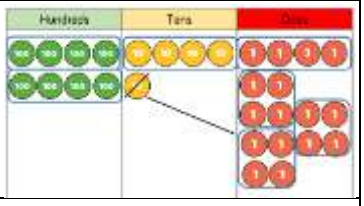
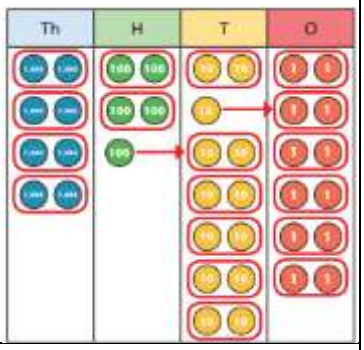
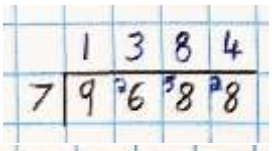
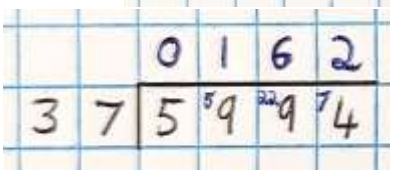
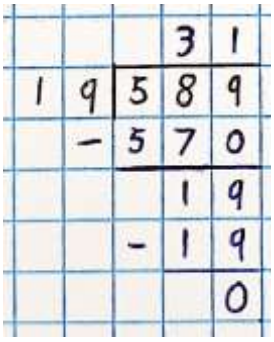
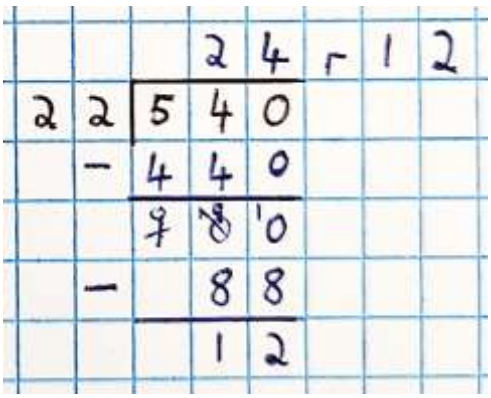
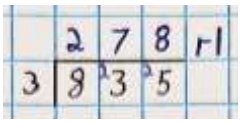
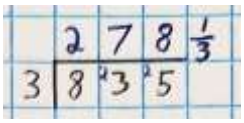
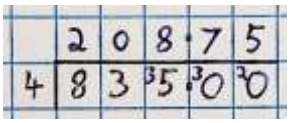
Division in Year 4

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Recall division facts for multiplication tables up to 12 × 12. Use place value, known and derived facts to divide mentally, including dividing by 1, and recognising and using factor pairs and commutativity in mental calculations.	groups of, grouping, sharing, share, shared, ___ each, in pairs, in ___s, equal, equally, same size, same amount, division, divide, divided by, dividing, divided into, left (over), array, guess, estimate, remainder, approximate, approximately, **commutative (law), commutativity, regroup, exchange**	**Concrete:**  Children will link division to multiplication by creating arrays and finding the number sentences that can be generated as a result, for example (right): 15 ÷ 3 = 5 15 ÷ 5 = 3 5 x 3 = 15 3 x 5 = 15 5 = 15 ÷ 3 3 = 15 ÷ 5 15 = 5 x 3 15 = 3 x 5 Children will also use base 10 representations to divide numbers and demonstrate the need for regrouping/exchanging (right).   When finding remainders, children will initially use objects to see how many are left over, for example when dividing 14 by 3 to find the quotient of 4 remainder 2 (left). **Pictorial:**  Children will use number lines to divide, as in the examples of 192 ÷ 8 = 24 and 92 ÷ 3 = 30 r 2 to the right. Note: Counting back on number lines reinforces the concept that the quotient will be smaller than the dividend, making the process of division distinct from the process of multiplication, however children can also count up on number lines rather than back, as long as their understanding of the process is fully secure before doing so. Children will also use bar models to represent division (right).  **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children in Year 4 will use the formal written method of short division to divide dividends of up to 3 digits by single-digit divisors:   In the event of remainders, children will record them using an r, as below: 

Division in Year 5

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Divide numbers mentally drawing upon known facts. Divide numbers up to 4 digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context. Divide whole numbers and those involving decimals by 10, 100 and 1,000. Solve problems involving division including using their knowledge of factors and multiples, squares and cubes. Solve problems involving multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving division, including scaling by simple fractions and problems involving simple rates.	groups of, grouping, sharing, share, shared, ___ each, in pairs, in ___s, equal, equally, same size, same amount, division, divide, divided by, dividing, divided into, left (over), array, guess, estimate, remainder, approximate, approximately, commutative (law), commutativity, regroup, exchange, **factor, multiple, square, cube, scale (by), scaling (by), simple fraction, decimal remainder, simple rate**	**Concrete:** Children will use place value counters to demonstrate the need for regrouping/exchanging (right).  **Pictorial:** Children will apply the same concept as above using pictorial representations of place value counters (right).  **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children in Year 5 will use the formal written method of short division to divide dividends of up to 4 digits by single-digit divisors (right):  In the event of remainders, children may record them either using an r, as a fraction or as a decimal (choosing the appropriate expression depending on the context) as below:   

Division in Year 6

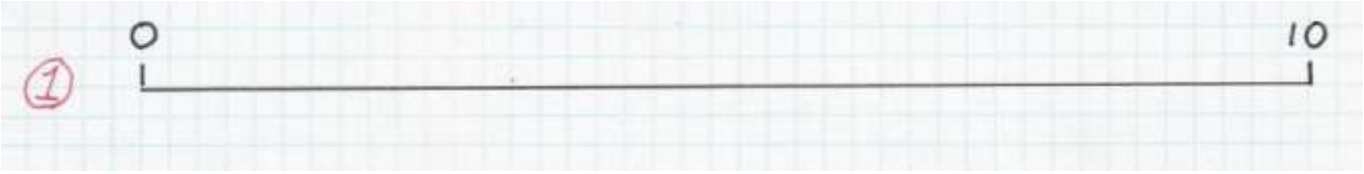
National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Divide numbers up to 4 digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. Solve problems involving division. Use written division methods in cases where the answer has up to two decimal places.	groups of, grouping, sharing, share, shared, ___ each, in pairs, in ___s, equal, equally, same size, same amount, division, divide, divided by, dividing, divided into, left (over), array, guess, estimate, remainder, approximate, approximately, commutative (law), commutativity, regroup, exchange, factor, multiple, square, cube, scale (by), scaling (by), simple fraction, decimal remainder, simple rate	**Concrete:** Children will use place value counters to demonstrate the need for regrouping/exchanging (right).  **Pictorial:** Children will apply the same concept as above using pictorial representations of place value counters (right).  **Abstract:** ***See Appendix 1 for further guidance on best practice.***  Children in Year 6 will use the formal written method of short division to divide dividends of up to 4 digits by single-digit divisors (above, right) and two-digit divisors (right).  Children will also use long division to divide by two-digit divisors:   In the event of remainders, children may record them either using an r, as a fraction or as a decimal (choosing the appropriate expression depending on the context).   

Appendix 1 - Notes on Best Practice with Written Methods

NCETM Guidance on Number Lines

(From https://www.ncetm.org.uk/features/five-tips-for-using-number-lines-in-key-stage-1/)

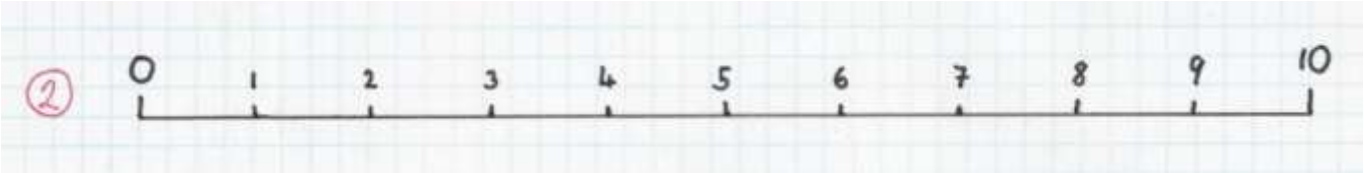
1. Finding the midpoint on a blank number line



Our first number line is a straight line with zero at one end and ten at the other. Could your pupils place the numbers one to nine on the straight line? It is likely numbers will be crowded together at either end of the line because the children have yet to develop the spatial reasoning skills needed. Asking pupils to find the midpoint (before asking them to place all the numbers) allows us to draw the children's attention to the fact that numbers are evenly spaced and to reinforce the concept of five as a midpoint.

A number line, or strip of paper folded in half, can be used to give children the opportunity to find the midpoint. Children can also create lines of different lengths on the playground with chalk or skipping ropes, and practise placing the class teddy on the midpoint. This activity provides a great opportunity to develop classroom talk and spatial language, such as 'in the middle' and 'equally spaced'.

2. Reasoning with a completed zero to ten number line



You could start this activity by revisiting the idea of the midpoint and ask children how they know what the midpoint is, providing a further opportunity for children to practise spatial language. However, the beauty of this number line is the opportunities it provides to fully explore the linear nature of the number system and to reason about the location of numbers within it.

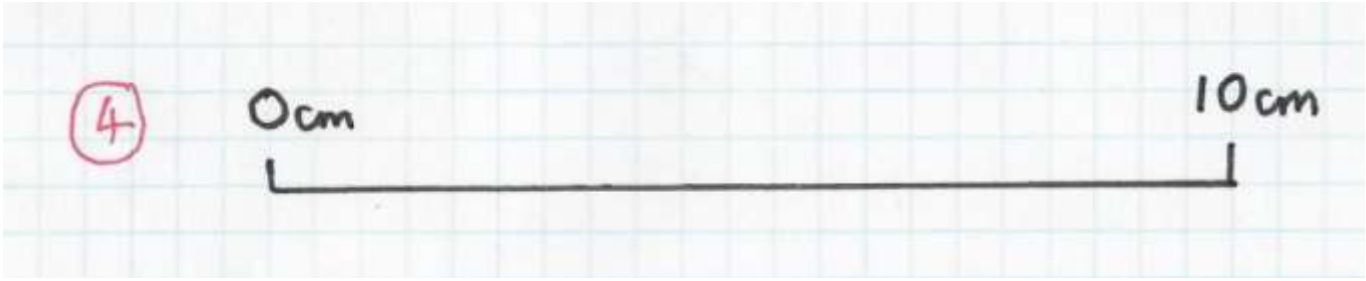
One way to start this exploration is through games of true or false using the inequality symbols; for example, seven is greater than eight. Can the children prove their answer using the number line? Another game that could be played is Guess My Number. Can children use clues about a number's location on the number line to find the number? These games bring the children back to the spatial language of 'greater than' and 'less than'. They are asked to justify their answers, whilst their attention is also drawn to where these numbers fit on the number line.

3. Placing numbers on a blank number line

Returning to the activity we started with, asking children to place the numbers onto the line allows us to revisit the concept of the midpoint and builds on our last activity. Children should have a better sense of being able to place the numbers evenly, but the main teaching point here is that they should be able to reason where their numbers sit in relation to each other.

You may ask them to compare their number line with a friend's number line, which provides an opportunity to ask which is better and why. Whose numbers are more evenly spaced? Have they correctly found the midpoint? At this stage, teachers can assess whether children are placing their number between or on the intervals.

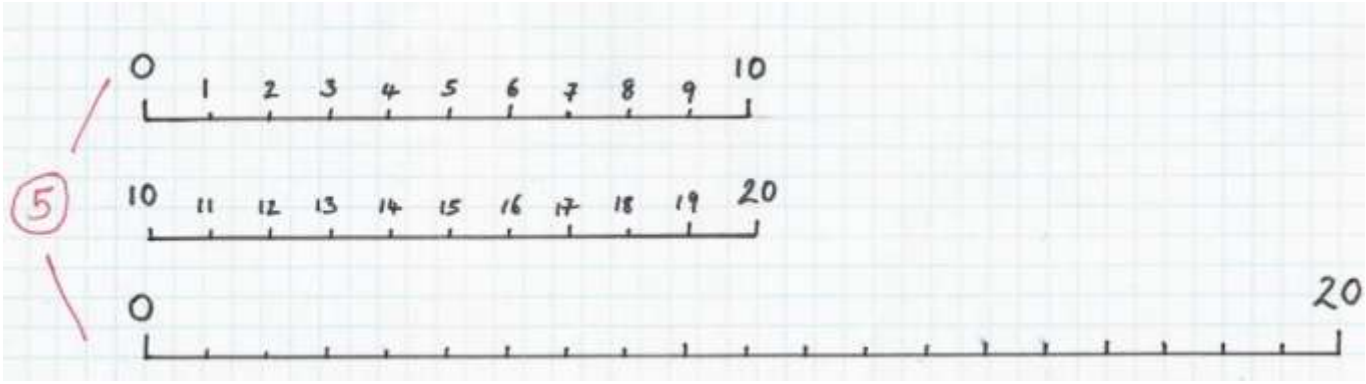
4. Using a number line as a measuring tool



This time we adjust the blank number line slightly by adding measures. Measure operates within our linear number system so a ruler is, in essence, a number line. Armed with their 10cm number line strip, can the children find objects shorter than 10cm or longer than 10cm?

By reminding children to line up their strip of paper so that 0cm lies at one end of the object, we are developing accuracy, which children will need when they start using a ruler. Can children find the midpoint? We can move on by asking them to find objects longer than 5cm or shorter than 10cm. We may even start to estimate and reason about where numbers fit in the linear system by asking where 7cm might be. Can children reason that it is between 5cm and 10cm? Where would they place it?

5. Comparing values on two completed number lines—zero to twenty



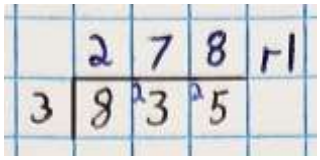
Comparing number lines encourages children to notice what is the same and what is different about numbers zero to 10 and numbers 10 to 20. With the number lines placed one on top of each other and lined up, ask children what they notice. Draw children's attention to the fact that where we have 1, on the next number line we have 10 and 1, which we call 11. Where we have 2, on the next number line we have 10 and 2, which we call 12 and so on. This enables children to see the relationship between the numbers that come after 10 and the numbers that come before 10 and the pattern of the number system.

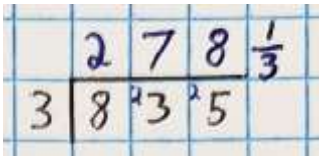
The children can discuss what is the same and what is different about the midpoints of both number lines. Will the midpoint always feature a five? By introducing a new number line, placed underneath, with the numbers zero to 20 placed at either end, children will identify that our midpoint is now 10.

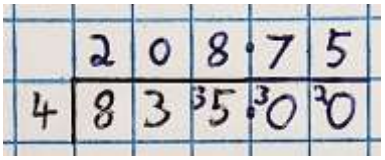
This activity enables children to explore '10 and a bit' numbers—the tricky teen numbers—and how they fit into the linear number system. This can be a 'wow' moment for children, as they start to make those connections, recognise those patterns, and begin to understand how they repeat into infinity.

Short Division

When children are doing short division, ensure that children still write one digit per square. Remainders may be expressed with the r in the same square as the first digit to save space, and fractions are best written small so that they fit inside one square. Ensure that children draw the division bracket using a ruler. There is no need for the division bracket to extend to accommodate the remainder unless continuing to divide beyond the decimal point, where there is more working out to do. Exchanges/regrouping are to be written as small, neat digits in the top-left corner of the next square as children work from left to right dividing each digit of the dividend by the divisor in turn, recording the quotient on top.

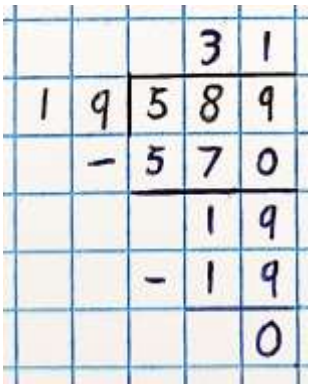


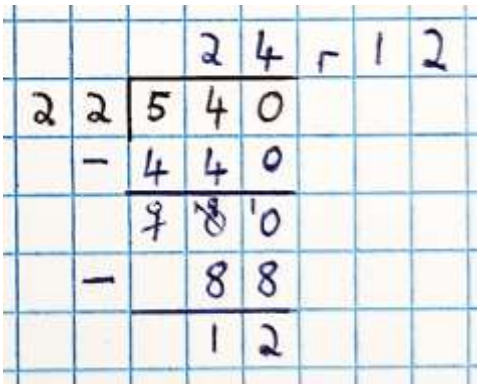




Long Division

When children are doing long division, all of the above still applies but there are some additional considerations. Children must include the minus sign and use a ruler to underline their subtractions, recording their subtraction within the calculation the same as they would with any other subtraction, showing their regrouping/exchanging in the same way.



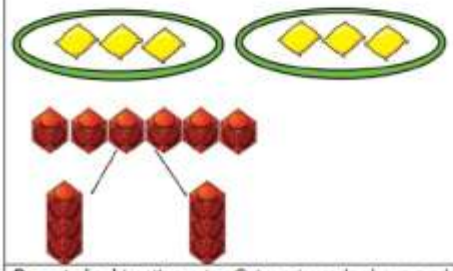
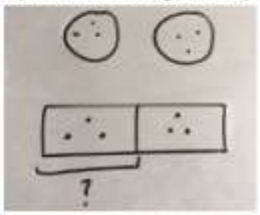
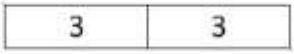
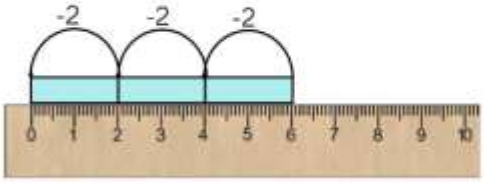
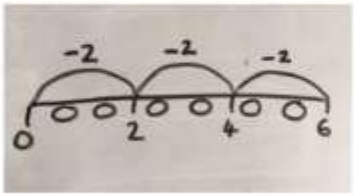
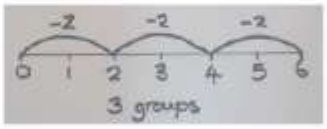
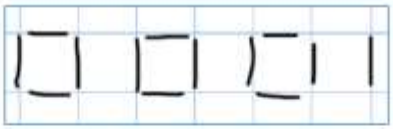
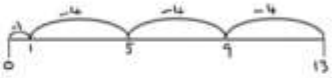
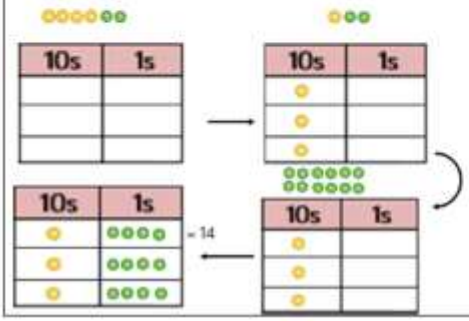
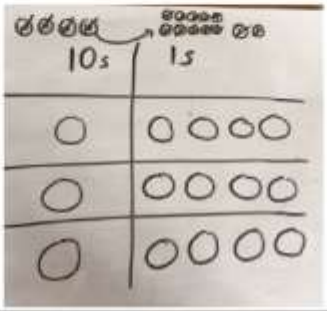


Appendix 2 – White Rose Calculation Guidance

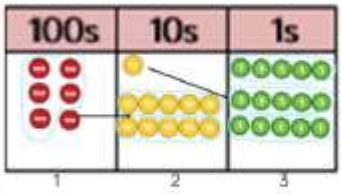
Below is a table taken from the White Rose Calculation guidance which shows how the concrete-pictorial-abstract approach can be used as a progression in learning of the concept of division.

Calculation policy: Division

Key language: share, group, divide, divided by, half.

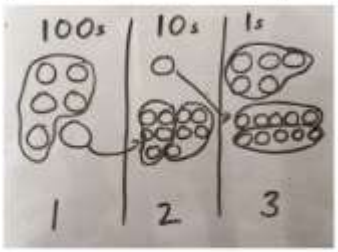
Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

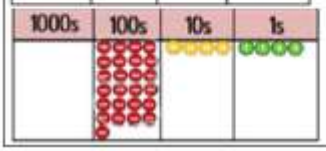


Children to the calculation using the short division scaffold.

$$5\overline{)615}\quad 123$$

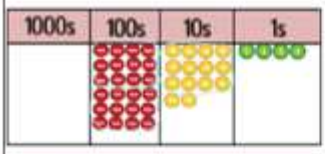
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



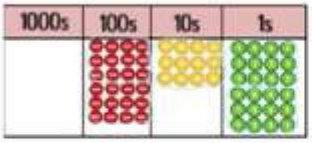
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

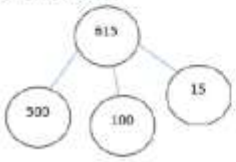


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5\overline{)615}$$

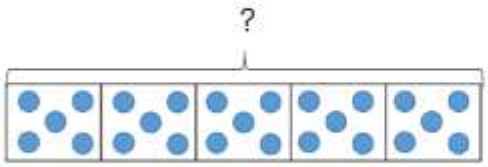
615 ÷ 5 =

☐ = 615 ÷ 5

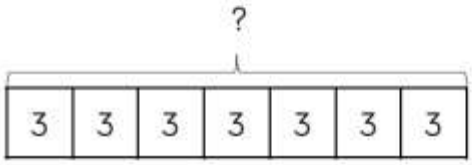
What is the calculation?
What is the answer?

Appendix 3 – White Rose Guidance on the Benefits of Different Representation Methodologies

Bar Model

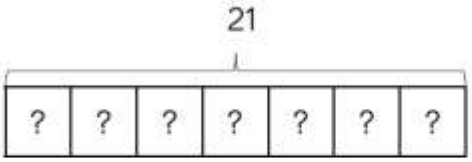


$5 \times 5 = 25$

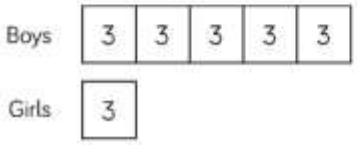


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

Number Shapes

$5 \times 4 = 20$

$4 \times 5 = 20$

$5 \times 4 = 20$

$4 \times 5 = 20$

$18 \div 3 = 6$

Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

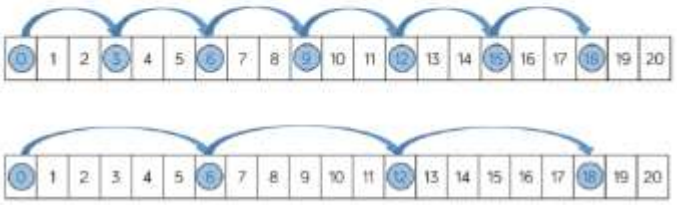
Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$6 \times 3 = 18$
$3 \times 6 = 18$

$18 \div 3 = 6$

Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

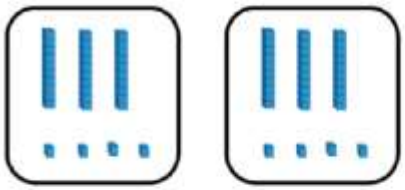
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

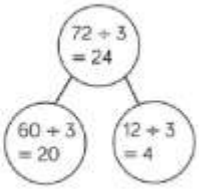
Base 10/Dienes (division)



$68 \div 2 = 34$

Tens	Ones

$72 \div 3 = 24$



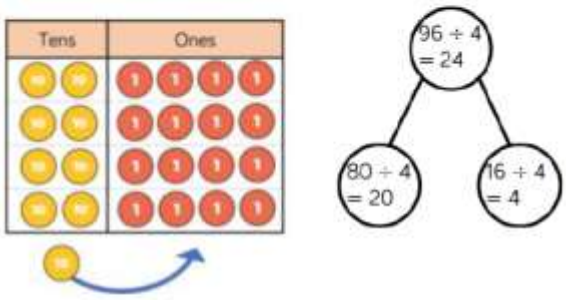
Benefits

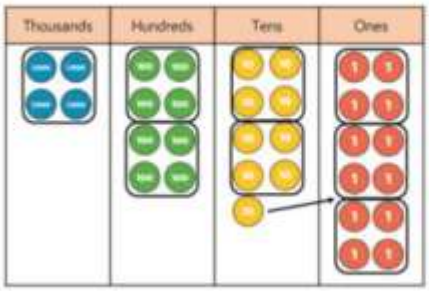
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (division)





$$\begin{array}{r} 1223 \\ 4\overline{)\,4892^{1}} \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

Appendix 4 – The Value of Precise Mathematical Vocabulary

The following are edited extracts from an article taken from RisingStars-UK.com at https://www.risingstars-uk.com/blog/november-2015/the-value-of-precise-mathematical-vocabulary in March 2024, written by Caroline Clissold.

For a comprehensive glossary, please see the separate document "NCETM Maths Glossary KS1-KS3" which is saved in PDF format with our calculation policies in the shared area.

For **addition**, precise terms include: **augend, add, addend, equal** and **sum**.
Augend is the amount that you start off with, addend is what you add to it and sum is the result. Augend comes from the Latin augendum, a thing to be increased.
Addend comes from the Latin word addendum which is an addition made to something.
Sum comes from the Latin word summa, which means highest.

The precise terms for **subtraction** are **minuend, subtract, subtrahend, equal** and **difference**.
Minuend stems from the Latin minuendus which means to be diminished or make smaller.
Subtrahend comes from the Latin subtrahendum which means to delete from a list or take away.
The **difference**, from the Latin word differentia meaning carrying away, is the result of the subtraction.

In **multiplication** the precise terms are: **multiplicand, multiplied by, multiplier, equal** and **product**.
Multiplicand comes from the Latin word multiplicandus which means to be increased or multiplied.
Multiplier is the number you are multiplying by and product is the result of the calculation.

Dividend, divided by, divisor, equals and **quotient** are precise terms for **division**.
Dividend comes from the Latin dividendum which is an amount to be divided into groups.
Divisor is the number by which another number is divided. Its original Latin word was divider.
Quotient comes from the Latin word quotiens which means 'how many times'.
The lines around such a calculation are called the **division bracket**, which makes a lot more sense.

Commutative is another word that children need to understand and begin to use.
Commutativity is an important part of addition and multiplication. If they understand this they would only need to learn half of their number facts and multiplication tables.

In **place value** it would be worth introducing the terms **positional, multiplicative** and **additive** to help the children understand these key areas of place value. It would also help to explain that our number system increases and decreases in **powers of 10**.

When dealing with **fractions** we need to be specific about the terms used when this area is introduced to children. Children should be introduced to **numerator** and **denominator** as correct vocabulary. The line that separates the two is a **vinculum**, which in Latin means 'bond'.