St Anne's Addition Calculation Policy

Date approved:	19th February, 2024
Date of review:	Summer Term 2025

Rationale:

This policy lays out the expectations for written calculations using addition and has been created to support the teaching of a mastery approach to mathematics in line with the National Curriculum and the White Rose scheme, which forms the framework of our curriculum through its long- and medium-term planning outline and small steps. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across EYFS, KS1 and KS2.

A Mastery Approach:

A mastery approach to learning involves the following five "big ideas" of effective maths teaching:

Coherence - a coherent learning progression offering deep and connected understanding

Representation and Structure concrete, pictorial and abstract representations are carefully structured to help pupils "see the maths"

Mathematical Thinking - looking for patterns and relationships, making connections, conjecturing, reasoning and generalising, communicating ideas using precise vocabulary

Fluency - - - efficient, accurate recall of key number facts and procedures, allowing pupils to move between different contexts and representations, choosing strategies

Variation - - - conceptual variation presents different representations of key features, while procedural variation presents different ways of proceeding through the learning journey (via scaffolding and support, etc)

Concrete – Pictorial – Abstract:

Mathematical understanding is developed through use of representations that are initially concrete (e.g. counters, multilink cubes, dienes, etc), and then pictorial (e.g. part-whole models, place value columns with images of counters in them, etc) to then facilitate abstract working (e.g. formal written methods).

This policy is a guide through an appropriate progression of representations. If at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate. As children move through the different stages, representations should be modelled alongside each other to ensure a secure understanding is maintained. Children should only move onto the abstract method when they have a secure understanding of the concept through an appropriate concrete and pictorial representation. This policy should be used in conjunction with the St Anne's Mathematics policy and St Anne's Mental Calculation policy. Teachers are also encouraged to refer to the NCETM Ready-To-Progress Criteria resources in ascertaining when children are ready to move on to new learning.

Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice guidance aimed at informing and enhancing teaching across all year groups. Many of these ideas come from the NCETM's Calculation Guidance document (published October 2015) and the White Rose Calculation Guidance (published in the academic year 2020-2021), which is intended to sit alongside a school's calculation policy.

Please note that while this list of representations is illustrative of the representations children will be exposed to and work with, it is not intended to be exhaustive and children will also encounter other representation methodologies that are not listed here.

Vocabulary:

Children will continually recap vocabulary learned in previous years to ensure that their understanding and usage of the terminology is fully developed, broad and specific in application. Vocabulary from previous years is included in each year group's columns in black, while new vocabulary that may not have been previously encountered is in green. Teachers are encouraged to check this list of vocabulary at the beginning and end of a relevant unit to ensure that they are modelling the full breadth and depth of vocabulary to the children, and that the children are using it in their verbal and written responses accurately and confidently.

Please see appendix 4 for notes on precise vocabulary, and for a comprehensive glossary, please see the separate document "NCETM Maths Glossary KS1-KS3" which is saved in PDF format with our calculation policies in the shared area.

Contents:

Addition in Reception

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Children will…

Count objects, actions and sounds.

Subitise.

Link the number symbol (numeral) with its cardinal number value.

Recall number bonds to 10.

Double numbers.

Find one more than a given number.

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add,
more,
altogether,
total,
double,
count up/on,
and,
make,
part,
whole

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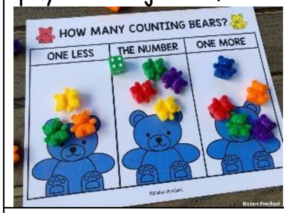
Concrete:

Counting all: a child finding 2+3 counts out 2 objects and then 3 objects, then finds the total by counting all of them together.

Counting on from the first number: a child finding 3+5 counts on from the first number: "four, five, six, seven, eight".

Children use bead strings, counters and physical objects for counting, as well as Numicon.



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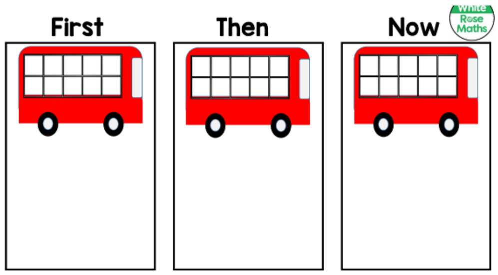
Pictorial:

Children count the totals of groups of pictures represented in different ways, including groups, tens frames, first, then and now processes, and dot groupings to support subitising.



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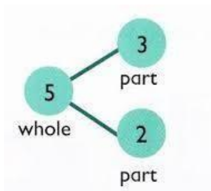
Abstract:

See Appendix 1 for further guidance on best practice.

Children relate the numbers of objects to numerals, and visualise this in pictograms and part-whole models:



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Addition in Year 1

<table>
<tr><th>National Curriculum
Objectives and Strategies</th><th>Vocabulary</th><th>Example Representation Methodologies</th></tr>
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Children will…

Read, write and interpret mathematical statements involving plus (+) and equals (=) signs.

Represent and use number bonds within 20.

Add 1-digit and 2-digit numbers to 20, including zero.

Solve 1-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = _ - 9 .

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add, more, altogether, total, double, count up/on, and, make, part, whole, plus, equals, equal to, number line, number bond

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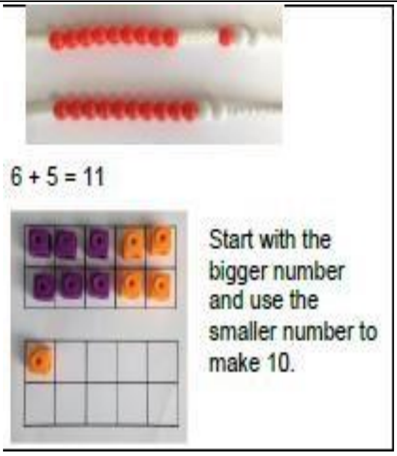
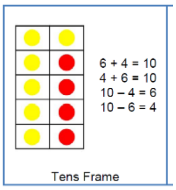
Concrete:

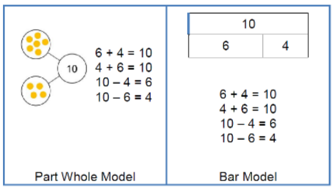


Children start with the larger number on the bead string and then count on using the smaller number 1-by-1 to find the total. This can also be done using counters on a tens frame.

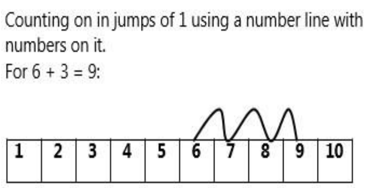


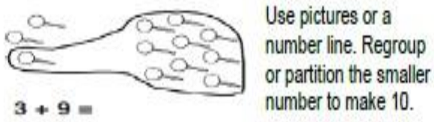
Pictorial:

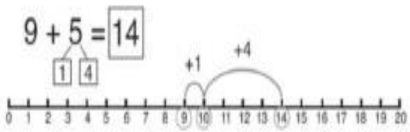
Children use part-whole models and bar models to visualise the structure of their total and addends.

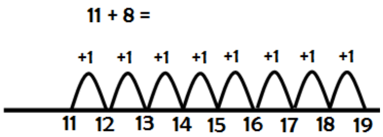


Children count on in 1s using a labelled number track or number line.



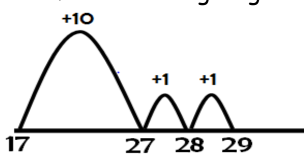






Children will also learn to add 10 on initially, to speed up the process and avoid mistakes that can arise from counting large numbers 1 by 1.

17 + 12 = 29



Abstract:

See Appendix 1 for further guidance on best practice.

5 + 12 = 17 Children hold the larger number in their memory, then count on using the smaller number to find the total. "12, 13, 14, 15, 16, 17"

Children read and write the plus and equals signs in number sentences, interpret addition number sentences and solve missing number problems, using concrete and/or pictorial representations if needed to help.

8 + 3 = ___ ___ + ___ = 6

7 + 5 = 7 + 3 + 2 = 12 "If I have seven, how many of 5 do I need to add to make 10? How many more do I still need to add on?"

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Addition in Year 2

National Curriculum
Objectives and Strategies

Children will…

Solve problems with addition using concrete objects and pictorial representations, including those involving numbers, quantities and measures.

Apply increasing knowledge of mental and written methods.

Recall and use addition facts to 20 fluently, and derive and use related facts up to 100.

Add numbers using concrete objects, pictorial representations, and mentally, including:
a 2-digit number and ones,
a 2-digit number and tens,
two 2-digit numbers, and
3 1-digit numbers.

Show that addition of two numbers can be done in any order (commutative).

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

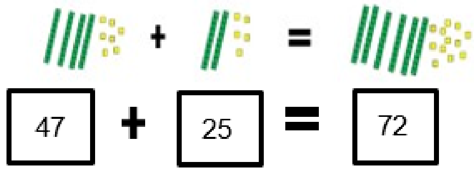
Vocabulary

add, more, altogether, total, double, count up/on, and, make, part, whole, plus, equals, equal to, number line, digit, tens, ones, greater than, less than, operation, partition, recombine, represents, inverse

Example Representation Methodologies

Concrete:

Children will use base-10 equipment (dienes) to represent tens and ones.

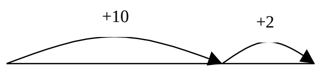


Pictorial:

It is valuable to use a range of representations (also see Y1). Continue to use number lines to develop understanding of:

Counting on in tens and ones:

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35

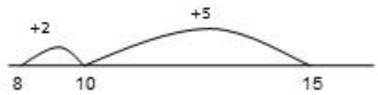


Partitioning and bridging through 10:

The steps in addition often bridge through a multiple of 10.

Children should be able to partition the 7 to relate adding the 2 and then the 5.

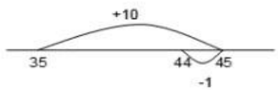
8 + 7 = 15



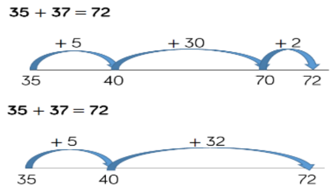
Adding 9 or 11 by adding 10 and adjusting by subtracting 1:

Add 9 by adding 10 and adjusting by subtracting 1.

35 + 9 = 44



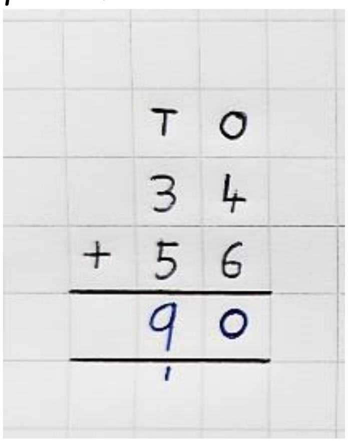
Use this method alongside manipulatives to ensure understanding. Use empty number lines, concrete equipment, hundred squares etc to build confidence and fluency in mental addition skills.



Abstract:

See Appendix 1 for further guidance on best practice.

Children will use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, for 2-digit numbers:



Children should not begin to use a compact column method before having a secure understanding of the process of addition, partitioning, regrouping/exchanging, and using place value columns.

Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

Addition in Year 3

<table>
<tr><th>National Curriculum
Objectives and Strategies</th><th>Vocabulary</th><th>Example Representation Methodologies</th></tr>
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Children will…

Solve problems, including missing number problems, using number facts, place value, and more complex addition.

Add numbers mentally, including:
a 3-digit number and ones,
a 3-digit number and tens, and
a 3-digit number and hundreds.

Add numbers with up to 3 digits, using formal written methods of columnar addition.

Estimate the answer to a calculation and use inverse operations to check answers.

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add, more, altogether, total, double, count up/on, and, make, part, whole, plus, equals, equal to, number line, digit, tens, ones, greater than, less than, operation, partition, recombine, represents, inverse, **hundreds, increase, expanded, digits, augend, addend, sum**

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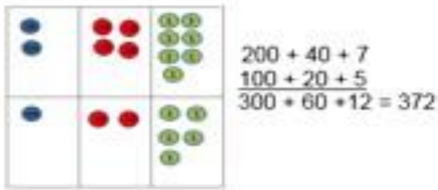
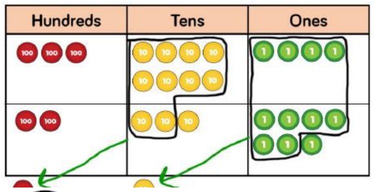
Concrete:

Introduce expanded column addition modelled with place value counters. (Dienes could be used for those who need a less abstract representation.)

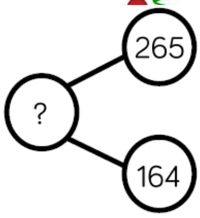


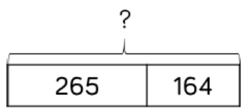
Pictorial:

After practically using the dienes and place value counters, children can draw the counters to help them to find sums.



Children will also use part-whole models and bar models to visualise addends and totals.

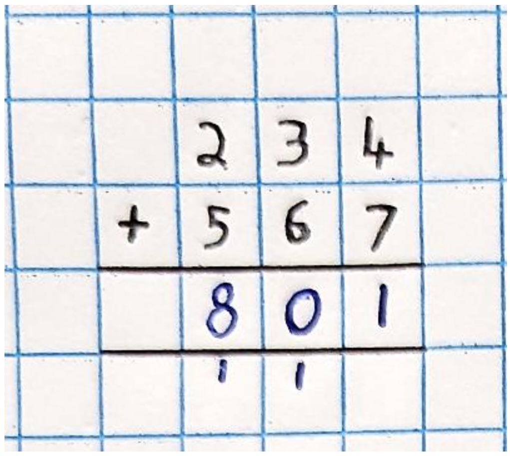




Abstract:

See Appendix 1 for further guidance on best practice.

Children will use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, for numbers of up to 3 digits:



Children should not begin to use a compact column method before having a secure understanding of the process of addition, partitioning, regrouping/exchanging, and using place value columns.

Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

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</table>

Addition in Year 4

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies

National Curriculum
Objectives and Strategies

Children will…

Add numbers with up to 4 digits using the formal written methods of columnar addition.

Estimate and use inverse operations to check answers to a calculation.

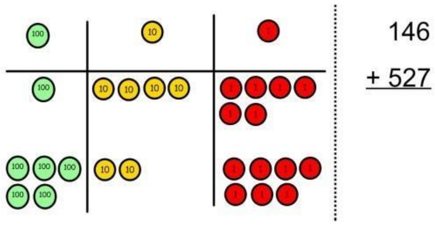
Solve addition two-step problems in contexts, deciding which operations and methods to use and why.

Vocabulary

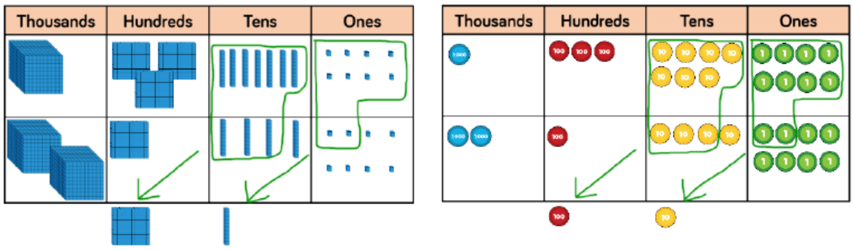
add, more, altogether, total, double, count up/on, and, make, part, whole, plus, equals, equal to, number line, digit, tens, ones, greater than, less than, operation, partition, recombine, represents, inverse, hundreds, increase, expanded, digits, augend, addend, sum, **thousands, decimal, decimal place, decimal point, tenths**

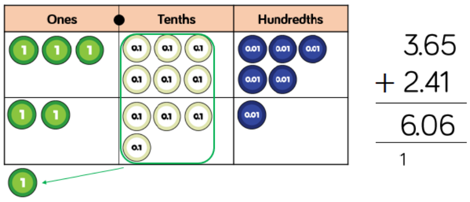
Example Representation Methodologies

Concrete:



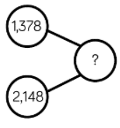
Children will use place value counters or dienes to reinforce understanding of the place value of each digit in addends and sums, including thousands and decimals, and of the process of regrouping/exchanging.





Pictorial:

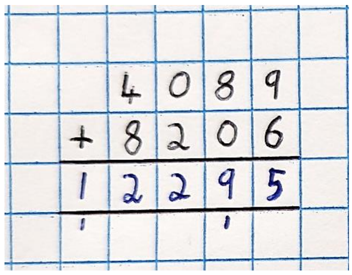
Children will use pictorial representations of the above methods to continue to develop their place-value understanding in addition. They will also continue to use part-whole models and number lines, learning to add on multiples of 10 initially to further increase the efficiency of the method.



Abstract:

See Appendix 1 for further guidance on best practice.

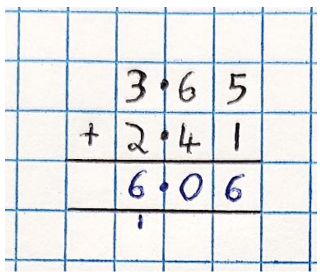
Children will use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, for numbers with up to 4 digits:



Children should not begin to use a compact column method before having a secure understanding of the process of addition, partitioning, regrouping/exchanging, and using place value columns.

Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

Children will also use compact column method for decimal numbers and to solve problems (e.g. in the context of money):



Addition in Year 5

National Curriculum
Objectives and Strategies

Children will…

Add whole numbers with more than 4 digits, including using formal written methods (columnar addition).

Add numbers mentally with increasingly large numbers.

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.

Solve problems involving numbers with up to three decimal places.

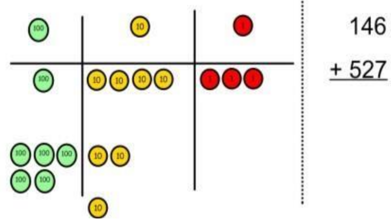
Vocabulary

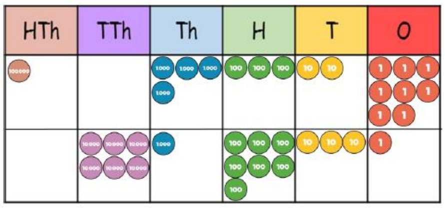
add, more, altogether, total, double, count up/on, and, make, part, whole, plus, equals, equal to, number line, digit, tens, ones, greater than, less than, operation, partition, recombine, represents, inverse, hundreds, increase, expanded, digits, augend, addend, sum, thousands, decimal, decimal place, decimal point, tenths

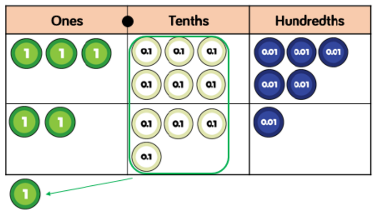
Example Representation Methodologies

Concrete:

Use concrete resources from prior year groups such as place value counters if they have not gained a secure understanding of addition.

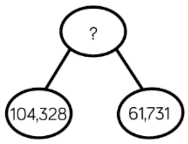


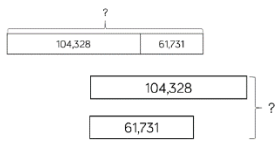


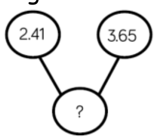


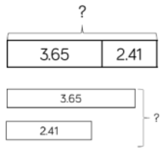
Pictorial:

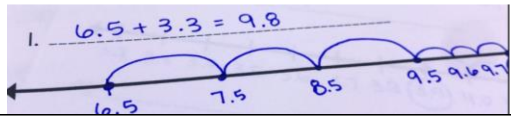
Use pictorial representations from prior year groups such as place value counters if they have not gained a secure understanding of addition with large numbers or decimals.







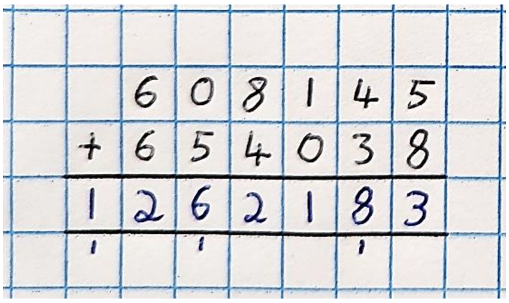




Abstract:

See Appendix 1 for further guidance on best practice.

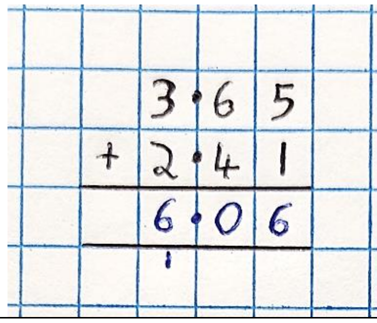
Children will use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, to add numbers with more than 4 digits:



Children should not begin to use a compact column method before having a secure understanding of the process of addition, partitioning, regrouping/exchanging, and using place value columns.

Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

Children will also use compact column method for decimal numbers and to solve problems (e.g. in the context of money):



Addition in Year 6

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Objectives and Strategies</th><th>Vocabulary</th><th>Example Representation Methodologies</th></tr>
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Children will…

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Perform mental calculations, including with mixed operations and large numbers.

Use their knowledge of the order of operations to carry out calculations involving the four operations.

Solve problems involving addition, subtraction, multiplication and division.

Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

</td><td>

add, more, altogether, total, double, count up/on, and, make, part, whole, plus, equals, equal to, number line, digit, tens, ones, greater than, less than, operation, partition, recombine, represents, inverse, hundreds, increase, expanded, digits, augend, addend, sum, thousands, decimal, decimal place, decimal point, tenths

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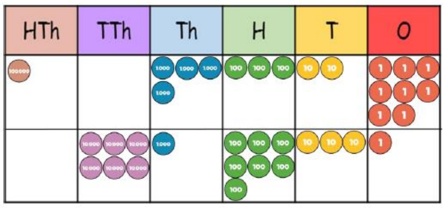
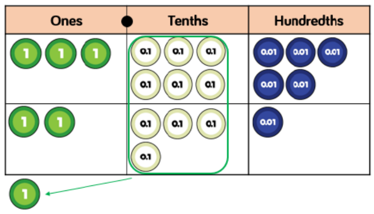
Concrete:

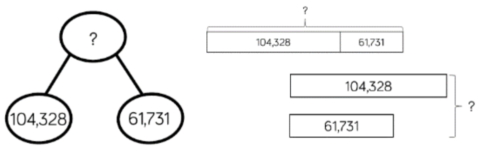
Use concrete resources from prior year groups such as place value counters if they have not gained a secure understanding of addition.

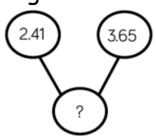


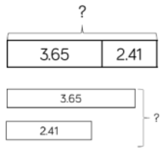


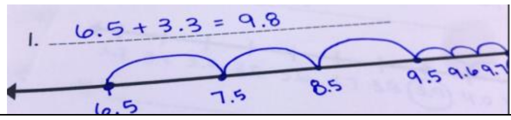
Pictorial:

Use pictorial representations from prior year groups such as place value counters if they have not gained a secure understanding of addition with large numbers or decimals.





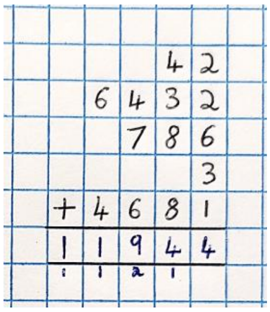




Abstract:

See Appendix 1 for further guidance on best practice.

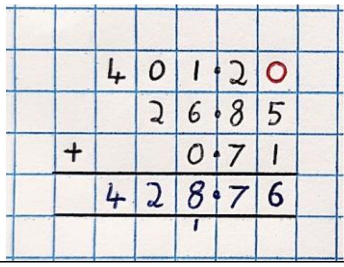
Children will use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, to add numbers with more than 4 digits and to find the sum of multiple addends (e.g. when problem-solving):



Children should not begin to use a compact column method before having a secure understanding of the process of addition, partitioning, regrouping/exchanging, and using place value columns.

Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

Children will also use compact formal written method to add decimals, adding in placeholder 0s to ensure clarity and consistency in method:



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Appendix 1 - Notes on Best Practice with Written Methods

NCETM Guidance on Number Lines

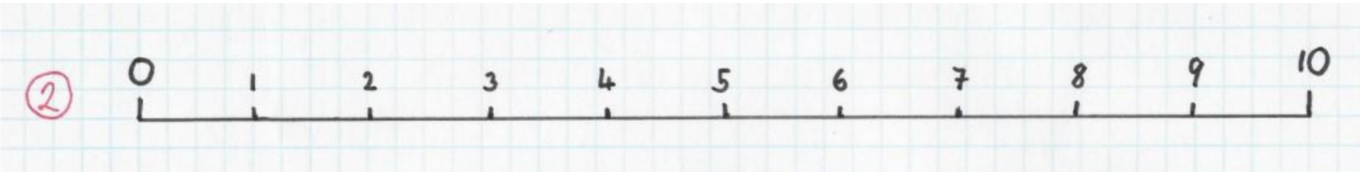
(From https://www.ncetm.org.uk/features/five-tips-for-using-number-lines-in-key-stage-1/)

1. Finding the midpoint on a blank number line

Our first number line is a straight line with zero at one end and ten at the other. Could your pupils place the numbers one to nine on the straight line? It is likely numbers will be crowded together at either end of the line because the children have yet to develop the spatial reasoning skills needed. Asking pupils to find the midpoint (before asking them to place all the numbers) allows us to draw the children's attention to the fact that numbers are evenly spaced and to reinforce the concept of five as a midpoint.

A number line, or strip of paper folded in half, can be used to give children the opportunity to find the midpoint. Children can also create lines of different lengths on the playground with chalk or skipping ropes, and practise placing the class teddy on the midpoint. This activity provides a great opportunity to develop classroom talk and spatial language, such as 'in the middle' and 'equally spaced'.

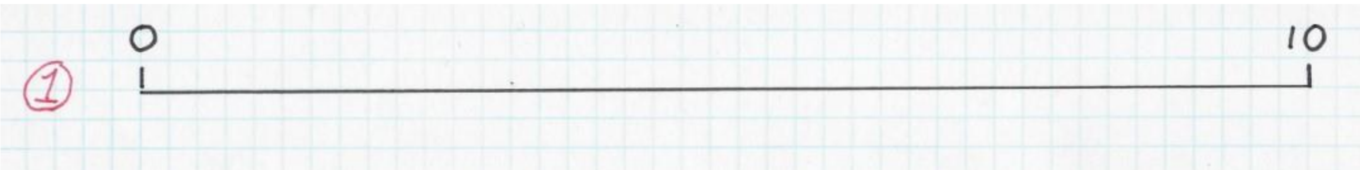
2. Reasoning with a completed zero to ten number line



You could start this activity by revisiting the idea of the midpoint and ask children how they know what the midpoint is, providing a further opportunity for children to practise spatial language. However, the beauty of this number line is the opportunities it provides to fully explore the linear nature of the number system and to reason about the location of numbers within it.

One way to start this exploration is through games of true or false using the inequality symbols; for example, seven is greater than eight. Can the children prove their answer using the number line? Another game that could be played is Guess My Number. Can children use clues about a number's location on the number line to find the number? These games bring the children back to the spatial language of 'greater than' and 'less than'. They are asked to justify their answers, whilst their attention is also drawn to where these numbers fit on the number line.

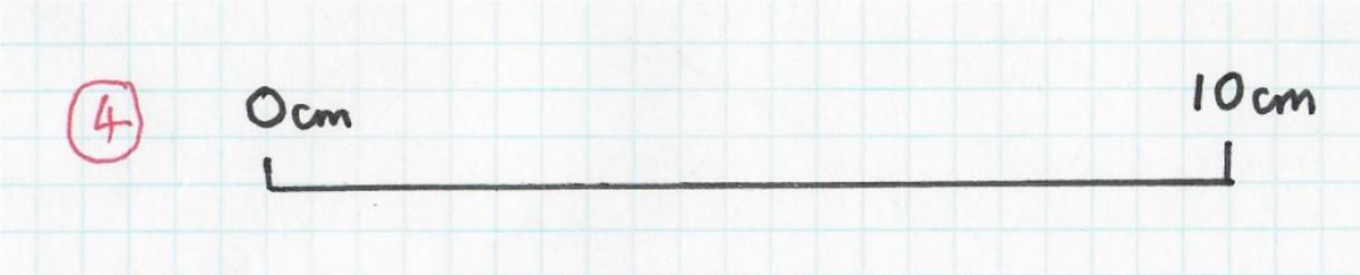
3. Placing numbers on a blank number line



Returning to the activity we started with, asking children to place the numbers onto the line allows us to revisit the concept of the midpoint and builds on our last activity. Children should have a better sense of being able to place the numbers evenly, but the main teaching point here is that they should be able to reason where their numbers sit in relation to each other.

You may ask them to compare their number line with a friend's number line, which provides an opportunity to ask which is better and why. Whose numbers are more evenly spaced? Have they correctly found the midpoint? At this stage, teachers can assess whether children are placing their number between or on the intervals.

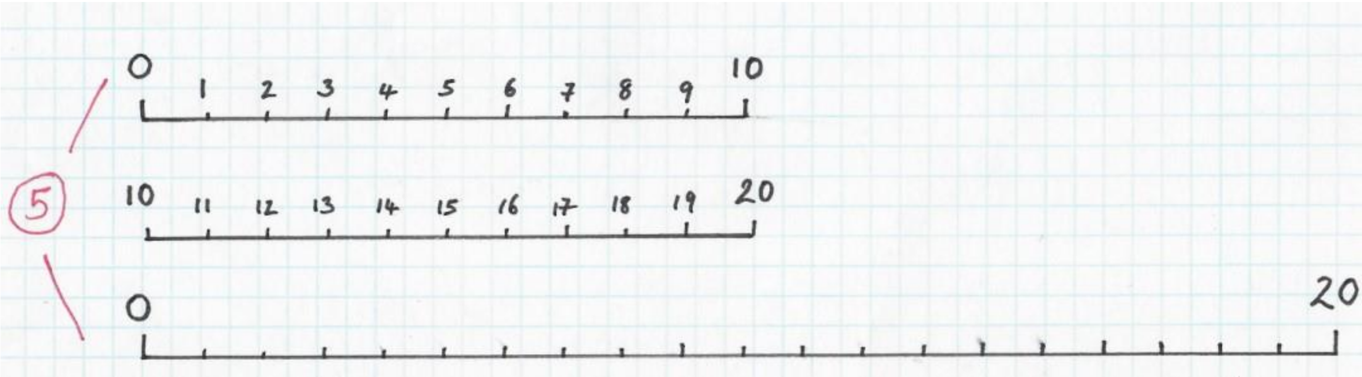
4. Using a number line as a measuring tool



This time we adjust the blank number line slightly by adding measures. Measure operates within our linear number system so a ruler is, in essence, a number line. Armed with their 10cm number line strip, can the children find objects shorter than 10cm or longer than 10cm?

By reminding children to line up their strip of paper so that 0cm lies at one end of the object, we are developing accuracy, which children will need when they start using a ruler. Can children find the midpoint? We can move on by asking them to find objects longer than 5cm or shorter than 10cm. We may even start to estimate and reason about where numbers fit in the linear system by asking where 7cm might be. Can children reason that it is between 5cm and 10cm? Where would they place it?

5. Comparing values on two completed number lines—zero to twenty



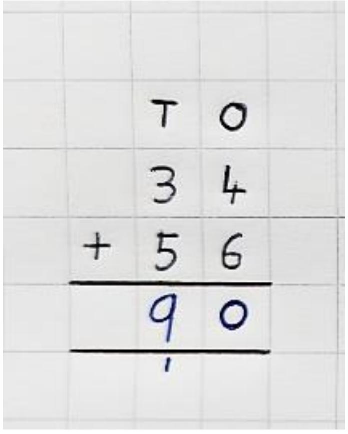
Comparing number lines encourages children to notice what is the same and what is different about numbers zero to 10 and numbers 10 to 20. With the number lines placed one on top of each other and lined up, ask children what they notice. Draw children's attention to the fact that where we have 1, on the next number line we have 10 and 1, which we call 11. Where we have 2, on the next number line we have 10 and 2, which we call 12 and so on. This enables children to see the relationship between the numbers that come after 10 and the numbers that come before 10 and the pattern of the number system.

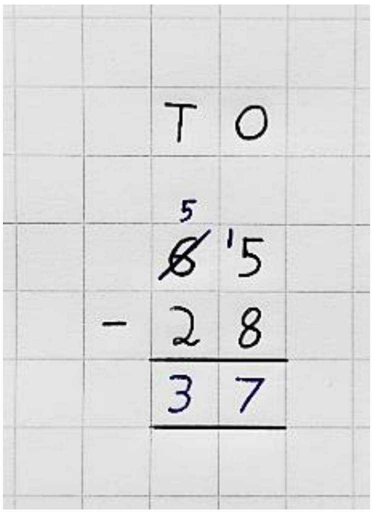
The children can discuss what is the same and what is different about the midpoints of both number lines. Will the midpoint always feature a five? By introducing a new number line, placed underneath, with the numbers zero to 20 placed at either end, children will identify that our midpoint is now 10.

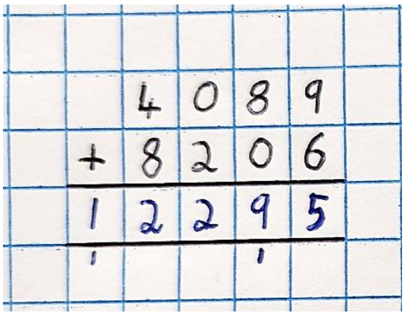
This activity enables children to explore '10 and a bit' numbers—the tricky teen numbers—and how they fit into the linear number system. This can be a 'wow' moment for children, as they start to make those connections, recognise those patterns, and begin to understand how they repeat into infinity.

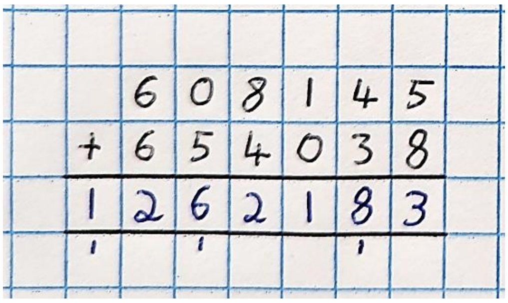
Column Methods

In KS1, it is best for children to always have the column headers (T and O) above the digits to ensure understanding. These can go directly above the digits in addition, whereas a space is needed between them in subtraction. In KS2, this is not usually necessary, but children who wish to add the headers can do so. This is not to be generally encouraged, as it slows down calculation time.

KS1: 



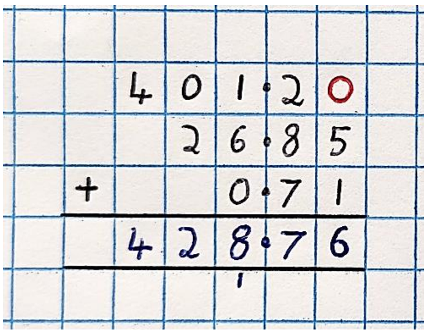
KS2: 



All digits are to be written with one digit per square. Ensure that the two addends are written with the ones columns aligned, and that the lines are drawn with a ruler. For addition, as the sum may be larger than the addends, always draw the lines one extra column wide. This reinforces the idea that addition results in a larger sum (in the case of positive integers). Ensure that children do not continue their lines past the ones column past where the decimal point would go – this is essential for avoiding misconceptions.

When writing carries (exchanges) underneath, ensure that children write very small, clear digits in the middle-top of the square in the appropriate column. This ensures that they do not confuse them with other digits in the method, and that we do not cause confusion in other methods in UKS2. A sharp pencil is essential for this. Encourage children to sharpen them habitually at the beginning of each lesson.

When using decimal numbers, the decimal point should always be placed in the centre of a line between two squares, not at the bottom and not in a square of its own on the page. This ensures that children do not treat it as an extra place value column between the ones and the tenths, and also that it is not confused with a full stop, as below. When addends have different numbers of decimal places, children should put in placeholder 0s to ensure consistency and avoid confusion.

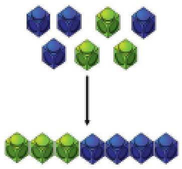
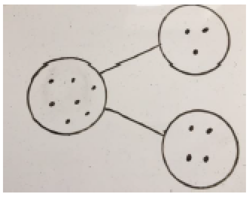
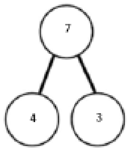
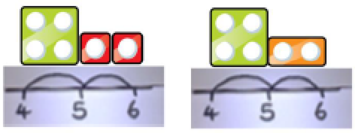
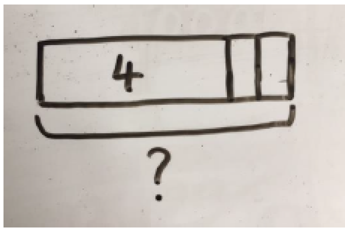
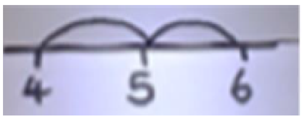


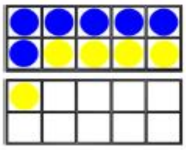
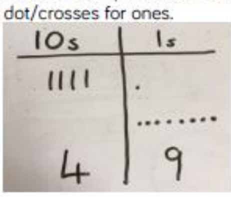
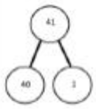
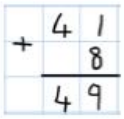
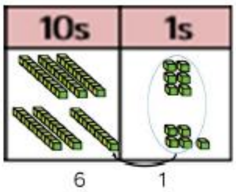
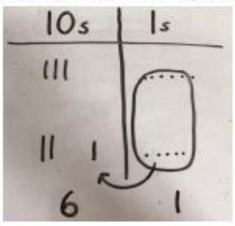
Appendix 2 – White Rose Calculation Guidance

Below is a table taken from the White Rose Calculation guidance which shows how the concrete-pictorial-abstract approach can be used as a progression in learning of the concept of addition.

Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

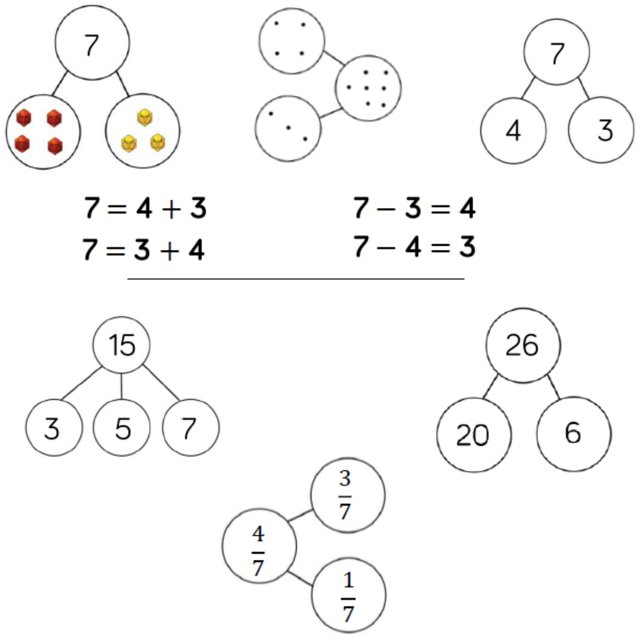
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.   	A bar model which encourages the children to count on, rather than count all.  	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.  	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  	Chidlren to represent the base 10 in a place value chart.  	Looking for ways to make 10. 36 + 25= 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1

Appendix 3 – White Rose Guidance on the Benefits of Different Representation Methodologies

The benefits of these methods explained by White Rose Mathematics (taken 2021 from White Rose addition calculation policy).

Part-Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

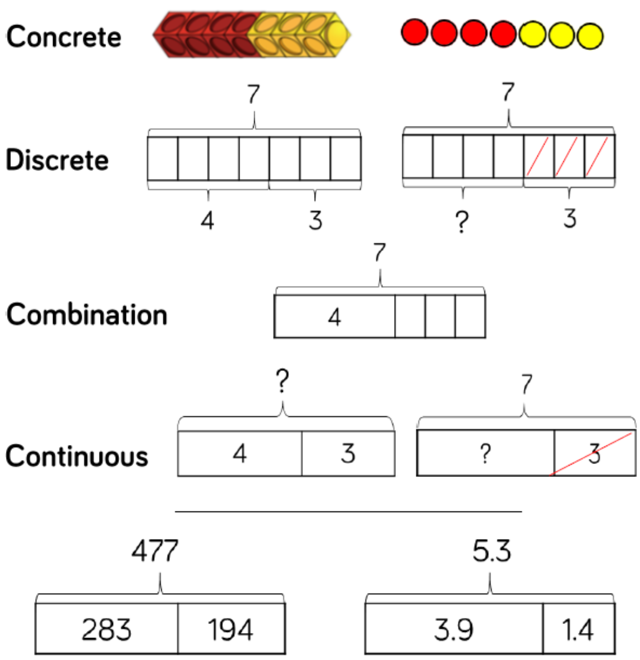
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

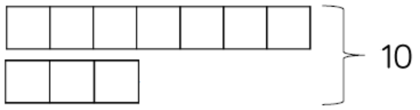
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

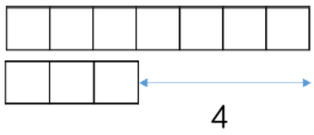
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete

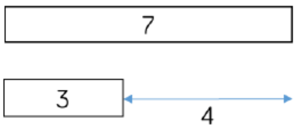


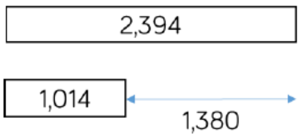
7 + 3 = 10



7 – 3 = 4

Continuous





7 – 3 = 4

2,394 – 1,014 = 1,380

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

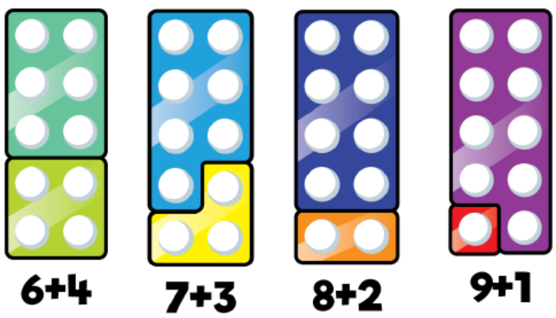
When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes

7 = 4 + 3

7 = 3 + 4

7 – 3 = 4



Benefits

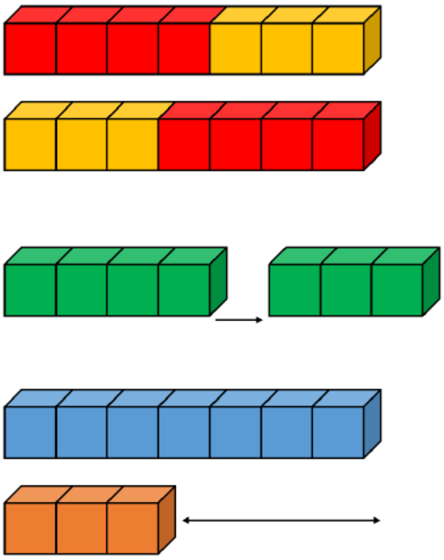
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Cubes



$7 = 4 + 3$

$7 = 3 + 4$

$7 - 3 = 4$

$7 - 3 = 4$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

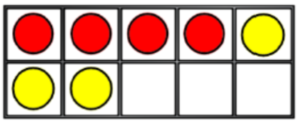
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

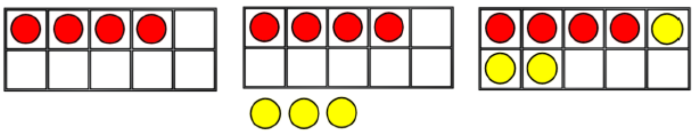
Ten Frames (within 10)



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

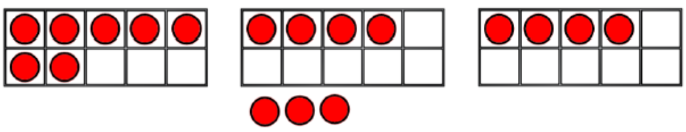
4 is a part.
3 is a part.
7 is the whole.

First | Then | Now



$4 + 3 = 7$

First | Then | Now



$7 - 3 = 4$

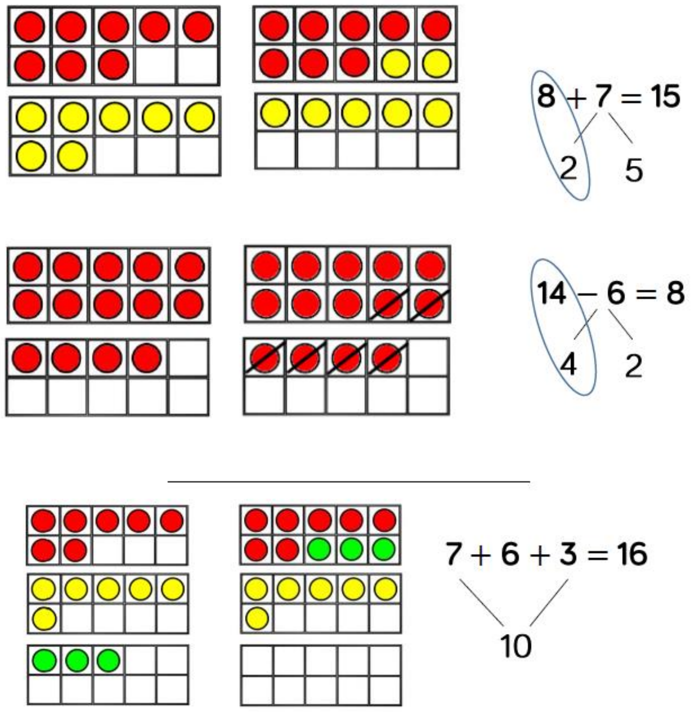
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



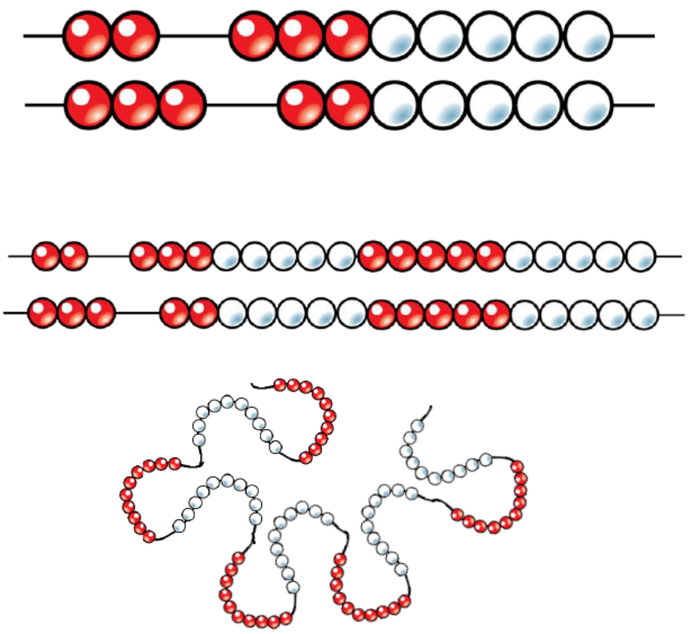
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

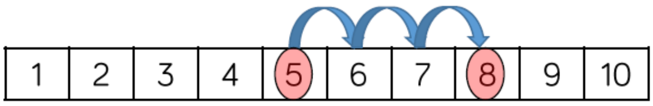
Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

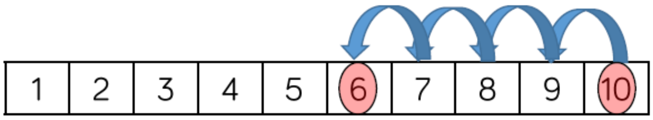
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks

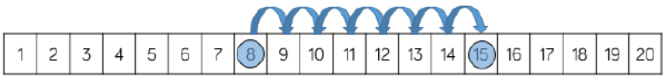
5 + 3 = 8



10 – 4 = 6



8 + 7 = 15



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

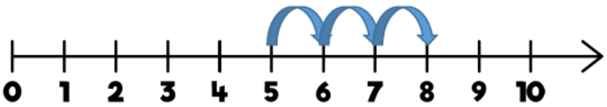
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

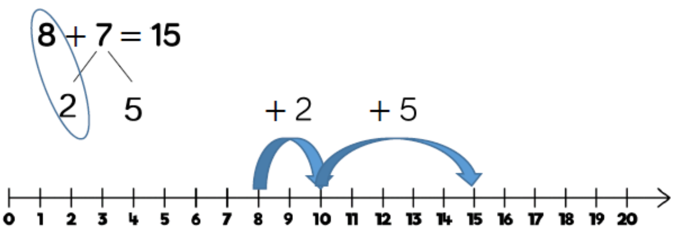
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

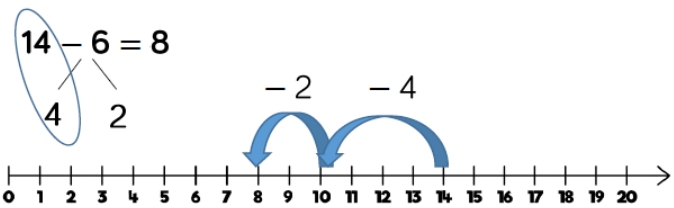
Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)

5 + 3 = 8







Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

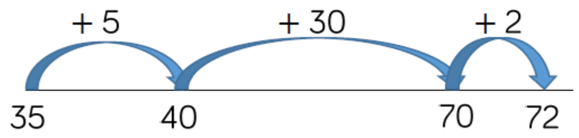
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

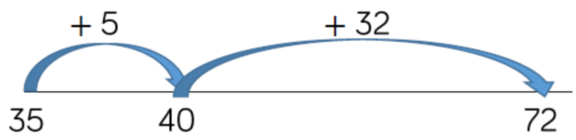
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

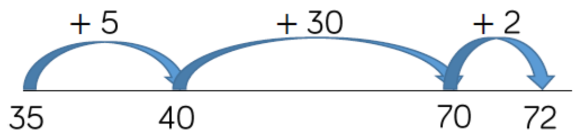
35 + 37 = 72



35 + 37 = 72



72 − 35 = 37



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

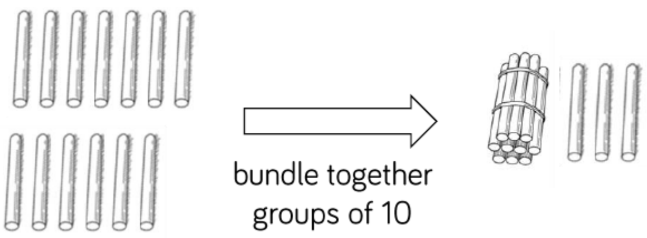
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

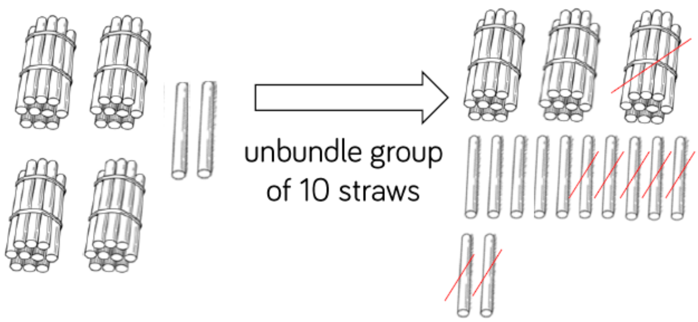
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Straws

7 + 6 = 13



42 − 17 = 25



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

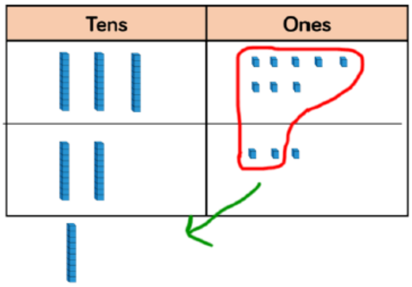
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

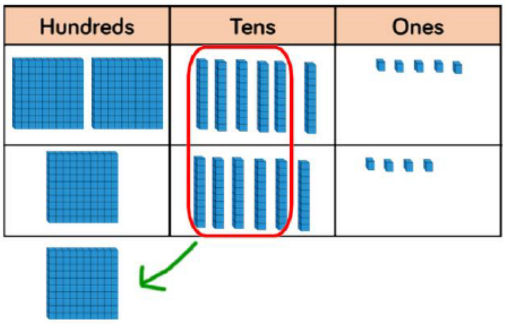
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)




```
  38
+ 23
----
  61
  1
```




```
  265
+ 164
-----
  429
   1
```

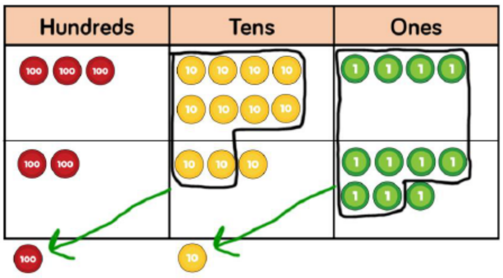
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

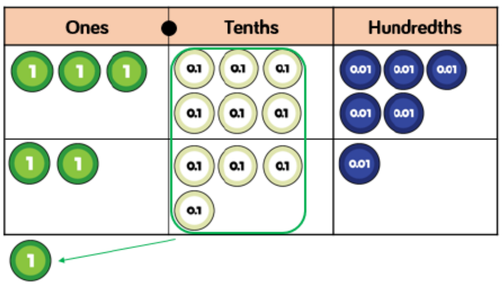
Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

Place Value Counters (addition)




```
  384
+ 237
-----
  621
  1 1
```




```
  3.65
+ 2.41
------
  6.06
  1
```

Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Appendix 4 – The Value of Precise Mathematical Vocabulary

The following are edited extracts from an article taken from RisingStars-UK.com at https://www.risingstars-uk.com/blog/november-2015/the-value-of-precise-mathematical-vocabulary in March 2024, written by Caroline Clissold.

For a comprehensive glossary, please see the separate document "NCETM Maths Glossary KS1-KS3" which is saved in PDF format with our calculation policies in the shared area.

For **addition**, precise terms include: **augend, add, addend, equal** and **sum**.
Augend is the amount that you start off with, addend is what you add to it and sum is the result. Augend comes from the Latin augendum, a thing to be increased.
Addend comes from the Latin word addendum which is an addition made to something.
Sum comes from the Latin word summa, which means highest.

The precise terms for **subtraction** are **minuend, subtract, subtrahend, equal** and **difference**.
Minuend stems from the Latin minuendus which means to be diminished or make smaller.
Subtrahend comes from the Latin subtrahendum which means to delete from a list or take away.
The **difference**, from the Latin word differentia meaning carrying away, is the result of the subtraction.

In **multiplication** the precise terms are: **multiplicand, multiplied by, multiplier, equal** and **product**.
Multiplicand comes from the Latin word multiplicandus which means to be increased or multiplied.
Multiplier is the number you are multiplying by and product is the result of the calculation.

Dividend, divided by, divisor, equals and **quotient** are precise terms for **division**.
Dividend comes from the Latin dividendum which is an amount to be divided into groups.
Divisor is the number by which another number is divided. Its original Latin word was divider.
Quotient comes from the Latin word quotiens which means 'how many times'.
The lines around such a calculation are called the **division bracket**, which makes a lot more sense.

Commutative is another word that children need to understand and begin to use.
Commutativity is an important part of addition and multiplication. If they understand this they would only need to learn half of their number facts and multiplication tables.

In **place value** it would be worth introducing the terms **positional, multiplicative** and **additive** to help the children understand these key areas of place value. It would also help to explain that our number system increases and decreases in **powers of 10**.

When dealing with **fractions** we need to be specific about the terms used when this area is introduced to children. Children should be introduced to **numerator** and **denominator** as correct vocabulary. The line that separates the two is a **vinculum**, which in Latin means 'bond'.